The Effect of the Interbank Network Structure on Contagion and Common Shocks

Abstract

This paper proposes a dynamic multi-agent model of a banking system with central bank. Banks optimize a portfolio of risky investments and riskless excess reserves according to their risk, return, and liquidity preferences. They are linked via interbank loans and face stochastic deposit supply. Comparing different interbank network structures, it is shown that money-center networks are more stable than random networks. Evidence is provided that the central bank stabilizes interbank markets in the short run only. Systemic risk via contagion is compared with common shocks and it is shown that both forms of systemic risk require different optimal policy responses.

Keywords: systemic risk, contagion, common shocks, multi-agent simulation

JEL classification: C63, E52, E58, G01, G21

1. Introduction

The recent financial crisis has shown that systemic risk takes many forms and is highly dynamic. It builds up slowly in normal times, and unwinds rapidly during times of distress. The insolvency of the US investment bank Lehman Brothers in September 2008 marked the tipping point between the build-up and rapid unwinding of systemic risks and led to a freeze in interbank markets. Banks were no longer able to obtain liquidity and engaged in costly fire sales. Central banks were forced to undertake unprecedented non-standard measures to ensure liquidity provision within the banking system.

This paper analyzes the non-trivial network structure of the bilateral interbank loans which form the money market. Interbank networks exhibit what Haldane (2009) describes as a *knife-edge*, or *robust-yet-fragile property*: in normal times the connections between banks lead to an enhanced liquidity allocation and increased risk sharing.[1] In times of crisis, however, the same interconnections can amplify initial shocks such as the insolvency of a large and highly interconnected bank.[2] This implies that there are two different regimes of financial stability: a stable regime in which initial shocks are contained, and a fragile regime in which initial shocks are transmitted via interbank linkages to a substantial part of the financial system. The knife-edge property of interbank markets can be attributed to a counterparty risk externality which is characteristic of over-the-counter markets (e.g. Acharya and Bisin (2010)). When a bank lends to a number of other banks it is oblivious to any links between those banks and might underestimate its portfolio correlation. A similar effect can be termed *correlation externality* and arises when a bank is oblivious to the asset holdings of other banks. The coun-

[1]See for example Allen and Gale (2000) (or Freixas et al. (2000) for a similar setting) who show that highly interconnected banking systems are less prone to bank-runs.

[2]The fragility of an interconnected financial system was analyzed by Gai and Kapadia (2010), who show that the risk of systemic crises is reduced with increasing connectivity on the interbank market. At the same time, however, the magnitude of such a crisis increases.

terparty risk externality can lead to interbank contagion (sometimes called cascading defaults), while the correlation externality can lead to common shocks.[3]

This poses the question of whether there exist network structures that are less prone to systemic risk (caused by either externality) and hence more resilient to financial distress. The massive intervention of central banks at the height of the financial crisis furthermore raises the question of whether central bank interventions can effectively stabilize interbank markets and ensure banks' liquitiy provision. Finally, in order to understand systemic financial fragility, it is necessary to compare the instabilities caused by the counterparty risk externality with instabilities caused by the correlation externality (i.e. to compare the effects of interbank contagion to the effects of common shocks).

This paper addresses the aforementioned questions by developing a simple dynamic model of a banking system that explicitly incorporates an evolving interbank network structure. Banks optimize a portfolio of risky investments and riskless excess reserves. Risky investments are long-term investment projects that fund an unmodelled firm sector while riskless excess reserves are short-term and held at the deposit facility of the central bank.[4] Banks face a stochastic supply of household deposits and stochastic returns from

[3]A common shock can affect banks who have become overly correlated as a consequence of a correlation externality.

[4]Alternatively, excess reserves could be held in form of highly liquid T-bills.

risky investments. This gives rise to liquidity fluctuations and initiates the dynamic formation of an interbank loan network. Banks, furthermore, have access to central bank liquidity if they can provide sufficient collateral.

Three key results are obtained. First, this model is used to compare different possible interbank network structures, and it is shown that in random graphs the relationship between the degree of interconnectivity and financial (in-)stability is non-monotonic. In times of distress, money center networks (which are typically found in reality) are seen to be more stable than purely random networks. In tranquil times, however, I show that different interbank network structures do not have a substantial effect on financial stability. The key intuition behind this behaviour is a regime switching property of the model financial system. In tranquil times, liquidity demand-driven interbank lending is low and cascading defaults are thus contained. In times of crisis, individual banks suffer larger liquidity fluctuations and engage in higher liquidity-driven interbank lending. This drives the financial system as a whole into a *contagious regime*. When exactly the regime switching behaviour occurs depends on the interbank network structure.

Second, I show that the central bank can stabilize the financial system in the short run. In the long run, however, the system always converges to a steady state which depends, amongst other things, on the interbank network structure. Central bank liquidity provision helps banks to withstand liquidity shocks for a longer time. This, however, allows banks that would otherwise be insolvent to engage in liquidity demand-driven interbank borrowing. The result is that the financial system as a whole is more highly interconnected and more likely to enter the contagious regime.

Third, I show that the introduction of a common shock hitting all banks simultaneously can cause substantial financial fragility but has a less severe impact on the liquidity provision of the interbank market. This finding is of particular importance for policymakers implementing emergency measures in times of a crisis: while interbank contagion requires mainly liquidity provision, a common shock requires banks to be recapitalized.

The remainder of this paper is organized as follows. After this introduction, section two outlines the contribution to the literature. Section three describes the dynamic model that has been used to analyze the aforementioned questions. Section four will present the main results, section five provides a discussion of further model implications, while sextion six concludes.

2. Relation to the Literature

The literature on financial networks has been growing rapidly over the past few years.[5] As a result, this paper relates to various strands of literature. First, it relates to a class of network models using static network structures and fixed balance sheets. In contrast to this literature the present paper models banks that optimize their balance sheet structure in every period and continuously adapt the interbank network structure. Closest to the present

[5] An overview of the existing literature can be found, for example, in Allen et al. (2010).

paper are the works by Iori et al. (2006) and Nier et al. (2008). In the model of Iori et al. (2006) banks' balance sheets consist of risk-free investments and interbank loans as assets, with deposits, equity and interbank borrowings as liabilities. Banks receive liquidity shocks via deposit fluctuations and pay dividends if possible. Nier et al. (2008) describe the banking system as a random graph where the network structure is determined by the number of banks and the probability that two nodes are connected. The banks' balance sheet consists of external assets investments and interbank assets on the asset side and net worth, deposits, and interbank loans as liabilities. Net worth is assumed to be a fixed fraction of a bank's total assets and deposits are a residual, designed to complete the bank's liabilities side. Idiosyncratic shocks that lead to a bank's default are distributed equally within the interbank market. Both papers assume a risk-free investment opportunity and Nier et al. (2008) further assume deposits to be residual. By contrast, I explicitly allow the possibility of risky investments and deposit fluctuations.

In a recent paper, Bluhm et al. (2012) develop an intertemporal agent-based model of banks with a dynamic interbank network. While Bluhm et al. (2012) focus on the contribution of individual banks to overall systemic risk, I analyze the impact of the interbank network structure on financial stability. Ladley (2011) finds in a *static* network setting that for small shocks, high interconnectivity helps to stabilize the system, while for large shocks high interconnectivity amplifies the initial impact. Such a static approach has been considered by a number of authors, including Gai and Kapadia (2010), Battiston et al. (2012), and, earlier, Eisenberg and Noe (2001). In contrast to this literature, I consider a *dynamic* contagion model where banks optimize their balance sheet structure and as a result the actual interbank network structure.

Second, this paper relates to the empirical literature on the topology of interbank networks by conducting a dynamic analysis of interbank contagion with general interbank network topologies. Such empirical analyses include Blåvarg and Nimander (2002), Boss et al. (2004), van Lelyveld and Liedorp (2006), Degryse and Nguyen (2007), and Becher et al. (2008). These papers show that interbank networks often exhibit a scale-free topology, i.e. they are characterized by a few money center banks with many interconnections and a large number of small banks with few connections.

Third, this paper contributes to a vast literature on systemic risk. A large part of the literature on systemic risk in interbank markets has focused on the analysis of contagion effects (i.e. studying the counterparty risk externality). Recently, more attention has been given to the correlation externality and the analysis of common shocks as sources of systemic risk. Acharya and Yorulmazer (2008), for example, point out how banks are incentivized to increase the correlation between their investments, and thus the risk of an endogenous common shock, in order to prevent costs arising from potential information spillovers.

Fourth, in addition to the existing literature on interbank networks, this paper introduces a central bank as a key player in the financial system. To motivate central bank interventions, Allen et al. (2009) and Freixas et al. (2010) show that central bank intervention can increase the efficiency of interbank markets. The present model investigates the effects of central bank intervention on contagion and common shocks.

3. The Model

This section develops a dynamic model of a banking system that can be used to analyze the impact of the interbank network structure on financial stability. First, deposit fluctuations have to be included: (i) Because of the maturity transformation that banks perform and since deposits usually have a short maturity, deposit fluctuations can lead to illiquidity. Banks which become illiquid have to liquidate their long-term investments at steep discounts. Due to marked-to-market accounting, these steep discounts will lead to losses in banks' trading books and have to be compensated by banking capital. Thus, illiquidity can lead to insolvency. (ii) As deposit fluctuations are generally considered to be one of the reasons why banks engage in interbank lending (see, for example, Allen and Gale (2000)), they have to be included in all models of systemic risk. Without deposit flucutations as a driving force behind the formation of interbank networks, it is impossible to describe the counterparty risk externality in a dynamic setting. Second, as fluctuations in investment returns have to be compensated by banking capital, risky investments are a major cause of bank insolvencies. Without risky investments, it is impossible to model the correlation externality as it arises precisely in a situation when the returns on risky assets of a number of banks have negative realizations at the same time. In order to model common shocks, risky investments thus have to be taken into account.

3.1. Balance Sheets

The balance sheet of a bank k holds risky investments I^k and riskless excess reserves E^k as assets at every point in (simulation-) time $t = 1 \dots \tau$. The investments of bank k have a random maturity[6] $\tau_I^k > 0$ and I assume that each bank finds enough investment opportunities according to its preferences. The bank refinances this portfolio by deposits D^k (which are stochastic and have a maturity of zero), from which it has to hold a certain fraction rD^k of required reserves at the central bank, fixed banking capital BC^k (which is assumed to be held in a highly liquid form), interbank loans L^k and central bank loans LC^k.[7] Interbank loans and central bank loans are assumed to have a maturity of $\tau_L^k = \tau_{LC}^k = 0$. The maturity mismatch between investments and deposits is the standard maturity transformation of commercial banks. Interbank loans can be positive (bank has excess liquidity) or negative (bank has demand for liquidity), depending on the liquidity situation of the bank at time t. The same holds for central bank loans, where the bank can use either the main refinancing operations to obtain loans, or the deposit facility to loan liquidity to the central bank. The balance sheet of the commercial bank therefore reads as

$$I_t^k + E_t^k = (1 - r)D_t^k + BC_t^k + L_t^k + LC_t^k \tag{1}$$

[6]Maturity τ implies that the asset matures in $\tau + 1$ update steps.

[7]Interbank loans expose a lending bank to counterparty risk. The results of this paper could be translated to any setting with direct counterparty risk, including credit default swaps. Other types of links include common asset holdings and have been addressed, for example, by Arinaminpathy et al. (2012) and Wagner (2011).

The interest rate for deposits at a bank is r^d and the interest rate for central bank loans is r^b. Note that there is no distinction between an interest rate for the lending and deposit facility and, therefore, the interest rate on the interbank market will be equal to the interest rate for central bank loans.

The banks decide on their portfolio structure and portfolio volume. A constant relative risk aversion (CRRA) utility function is assumed to model the bank's preferences:

$$u^k = \frac{1}{1-\theta^k}\left(V^k(1+\lambda^k\mu^k - \frac{1}{2}\theta^k(\lambda^k)^2(\sigma^k)^2)\right)^{(1-\theta^k)} \tag{2}$$

where λ^k is the fraction of the risky part of the portfolio.[8] μ^k is the expected return, $(\sigma^k)^2$ the expected variance of the portfolio and θ^k is the banks risk aversion parameter.[9] $V_t^k = I_t^k + E_t^k$ denotes the bank's portfolio volume. The risky part of the portfolio follows from utility maximisation and reads as

$$(\lambda^k)^* = \min\left\{\frac{\mu^k}{\theta^k(\sigma^2)^k}, 1\right\} \quad \in [0,1] \tag{3}$$

The portfolio volume can be obtained by similar measures as

$$(V^k)^* = \left[\frac{1}{r^b}\left(\left(1+\lambda^k\mu^k - \frac{1}{2}\theta^k(\lambda^k)^2(\sigma^2)^k\right)^{(1-\theta^k)}\right)\right]^{1/\theta^k} \tag{4}$$

where r^b denotes the refinancing cost of the portfolio. Since banks obtain financing on the interbank market and from the central bank at the same

[8]This utility function can be scaled by a normalization parameter ξ which was taken to be $\xi = 1$ for simplicity, as it does not change any of the obtained results.

[9]Banks' risk aversion is drawn from a uniform distribution at the beginning of each simulation and kept constant throughout the simulation. A possible extension is to allow banks to adjust their risk aversion each period, based on current market conditions.

interest rate, this refinancing cost is equal to the main refinancing rate. It is possible to introduce a spread between the lending and deposit facilities and therefore allow the interest rate on the interbank market to vary stochastically around the main refinancing rate. If a bank now plans its optimal portfolio volume, it calculates with a planned refinancing rate. This refinancing rate follows from the bank's plan concerning how much in interbank loans it wants to obtain on the interbank market at a planned refinancing rate and how much in central bank loans it plans to obtain at the main refinancing rate. If this plan cannot be realized (for example if a bank's liquidity demand is unsatisfied on the interbank market), banks make a non-optimal portfolio choice. This possibility is ruled out for the sake of simplicity. Note that a market for central bank money is not explicitely modelled. The central bank instead accommodates all liquidity demands of commercial banks, as long as they can provide the neccessary securities. This assumption is not unrealistic in times of crises, as is shown, for example, by the full allotment policy of the European Central Bank at the peak of the crisis.

3.2. Update Algorithm

In the simulation I have implemented an update algorithm that determines how the system evolves from one state to another. The algorithm is divided into three phases that are briefly described here. Every update step is done for all banks for a given number of sweeps. At the beginning of phase 1 the bank holds assets and has liabilities from the end of the previous period:

$$\underline{I}^k_{t-1} + \underline{E}^k_{t-1} + r\underline{D}^k_{t-1} = \underline{D}^k_{t-1} + \underline{BC}^k_{t-1} + \underline{L}^k_{t-1} + \underline{LC}^k_{t-1} \tag{5}$$

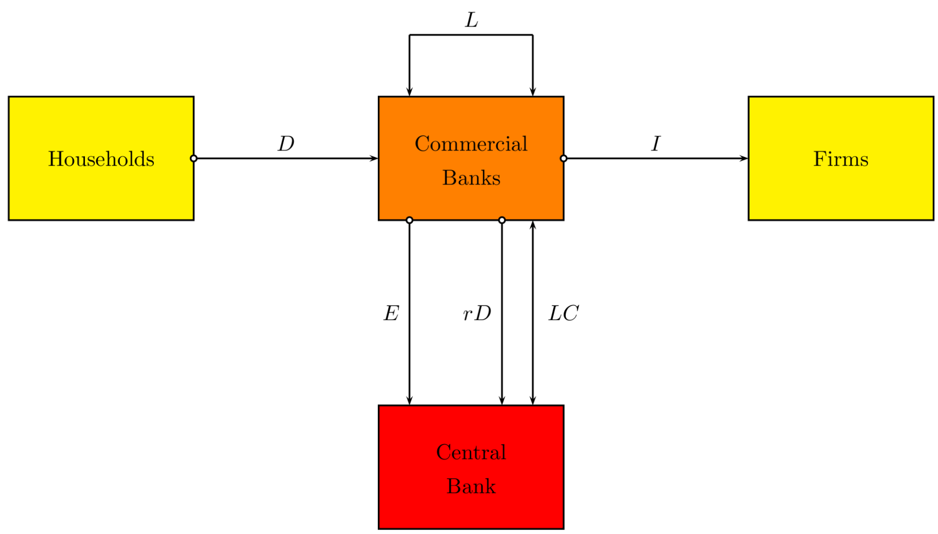


Figure 1: Interaction dynamics of the model. The private sector (household/firms), the banking sector (commercial banks) and the central bank interact via the exchange of deposits, investments, loans, excess- and required reserves and central bank loans. Arrows indicate the direction of fund flows.

where an underline denotes realized quantities. In period 0 all banks are endowed with initial values. The update step starts with banks obtaining the required reserves $r\underline{D}_{t-1}^{k}$ and excess reserves $\underline{E}_{t-1}^{k}$ plus interest payment from the central bank (it is assumed that, for both required and excess reserves, an interest of r^{b} is paid). The banks obtain a stochastic return for all investments $\underline{I}_{t-1}^{k}$ which might be either positive or negative. The firms furthermore pay back all investments $\underline{I}_{f}^{k}$ that were made in a previous period and have a maturity of $\tau_{I}^{k} = 0$. The banks then pay interest on all deposits that were deposited in the previous period. After that, the banks can either receive further deposits from the households, or suffer deposit withdrawings ΔD_t^k. At the end of the first period, all interbank and central bank loans plus interests are paid either to or by bank k.

At the beginning of phase 2, the bank's liquidity $\widehat{Q}^k$ is therefore given as:

$$\begin{aligned}\widehat{Q}_t^k \quad = \quad & (1+r^b)\left[r\underline{D}_{t-1}^k + \underline{E}_{t-1}^k\right] + \mu^k \underline{I}_{t-1}^k + \underline{I}_f^k - r^d \underline{D}_{t-1}^k \pm \Delta D_t^k \\ & -(1+r^b)\left[\underline{L}_{t-1}^k + \underline{LC}_{t-1}^k\right]\end{aligned} \tag{6}$$

All banks with $\widehat{Q}_t^k < 0$ are marked as illiquid and removed from the system. Banks that pass the liquidity check now have to pay required reserves $r\underline{D}_t^k$ to the central bank.

In phase 3 the bank k determines its planned level of investment $I_t^k = (\lambda^k)^*(V^k)^*$ and excess reserves $E_t^k = (1-(\lambda^k)^*)(V^k)^*$ according to equations (3) and (4). From this planned level and the current level of investments (all investments that were done in earlier periods and have a maturity $\tau_I^k > 0$), as well as the current liquidity (6) the bank determines its liquidity demand (or supply). If a bank has a liquidity demand, it will go first to the interbank market, where it asks all banks i that are connected to k (denoted as $i : k$) in a random order whether they have a liquidity surplus. If this is the case, the two banks will interchange liquidity via an interbank loan. The convention is adopted that a negative value of L denotes a demand for liquidity and therefore the interbank loan demand of bank k is given by

$$L_t^k = \widehat{Q}_t^k - I_t^k \tag{7}$$

From this, the realized interbank loan level can be obtained via the simple rationing mechanism:

$$\underline{L}_t^k = \min \begin{Bmatrix} L_t^k & , & -\sum_{i:k} L_t^i & \mid & L_t^i \cdot L_t^k < 0 & ; & \text{if } L_t^k > 0 \\ -L_t^k & , & \sum_{i:k} L_t^i & \mid & L_t^i \cdot L_t^k < 0 & ; & \text{if } L_t^k < 0 \end{Bmatrix} \tag{8}$$

Now, there are three cases, depending on the bank's liquidity situation. If a bank has neither a liquidity demand nor excess liquidity, it will not interact with the central bank and this step is omitted. However, if the bank still has a liquidity demand, it will ask for a central bank loan:

$$LC_t^k = \underline{L}_t^k - L_t^k \tag{9}$$

The central bank then checks whether the bank has the neccessary securities and, if so, it will provide the loan:

$$\underline{LC}_t^k = \max\left(LC_t^k, -\alpha^k \underline{I}_{t-1}^k\right) \tag{10}$$

where $\alpha^k \in [0,1]$ denotes the fraction of investments of bank k that are accepted as collateral by the central bank. If a bank has insufficient collateral, the central bank will not provide the full liquidity demand and the bank has to reduce the planned investment and excess reserve level. If the bank has no securities (no investments $\underline{I}_{t-1}^k$), it cannot borrow from the central bank. This rationing mechanism maps planned investment levels to realized ones.

The second case is that a bank has a large liquidity surplus even if all planned investments can be realized. In this case, the bank is able to pay dividends A_t^k and the dividend payment is determined by

$$A_t^k = \min\left\{LC_t^k, \beta^k \underline{I}_t^k\right\} \tag{11}$$

where $\beta^k \in [0,1]$ is the dividend level of bank k. The dividend level will typically be very close to 1 as shareholders will push the bank to pay dividends rather than use the money to deposit it at the central bank at low interest rates. The remaining:

$$\underline{LC}_t^k = LC_t^k - A_t^k \tag{12}$$

is transferred to the central bank's deposit facility. Finally the realized investments are transferred to the firm sector and the realized excess reserves are transferred to the central bank.

These steps are done for all $k = 1 \dots N$ banks in the system for $t = 1 \dots \tau$ time steps. As there are two stochastic elements in the simulation (the return of investments and the deposit level), two channels for a banks insolvency are modelled. The first channel is via large deposit withdrawals. As deposits are very liquid and investments are illiquid for a fixed, but random investment period, this maturity transformation might lead to illiquidity and therefore to insolvency. The second channel for insolvency is via losses on investments. If the banks banking capital is insufficient to cover losses from a failing investment, this bank will be insolvent. If a bank fails, all the banks that lent to this bank will suffer losses, which they have to compensate with their own banking capital. This is a possible contagion mechanism, where the insolvency of one bank leads to the insolvency of other banks which would have survived if it had not been for the first bank's insolvency. The impact of the contagion effect will depend on the precise network structure of the interbank market at the time of the insolvency.

3.3. Network theory

A financial network consists of a set of banks (nodes) and a set of relationships (edges) between the banks. Even though many relationships exist between banks, this paper focuses on relationships that stem from interbank lending. For the originating (lending) bank the loan will be on the asset side of its balance sheet, while the receiving (borrowing) bank will hold the loan as a liability. To describe the topology of a network, some notions from graph theory are helpful. The starting point is the definition of a graph.

Definition 1. *A (un)directed graph $G(V, E)$ consists of a nonempty set V of vertices and a set of (un)ordered pairs of vertices E called edges. If i and j are vertices of G, then the pair ij is said to join i and j.*

Graphs are sometimes referred to as networks and the two terms are used interchangeably. Since the focus of this paper is on interbank markets, the nodes of a network are (commercial) banks and the edges are interbank loans between two banks. For every graph, a matrix of bilateral exposures describing the exposure of bank i to bank j can be constructed.

Definition 2. *The matrix of bilateral exposures $W(G) = [w_{ij}]$ of an interbank market G with n banks is the $n \times n$ matrix whose entries w_{ij} denote bank i's exposure to bank j. The assets a_i and liabilities l_i of bank i are given by $a_i = \sum_{j=1}^{n} w_{ij}$ and $l_j = \sum_{j=1}^{n} w_{ji}$.*

Closely related to the matrix of bilateral exposures is the adjacency matrix that describes the structure of the network without referring to the details of the exposures.

Definition 3. *The entries a_{ij} of the adjacency matrix $A(G)$ are 1 if there is an exposure between i and j and zero otherwise.*

The interconnectedness of a node can be defined as the in- and out-degree of the node.

Definition 4. *The in-degree $d_{in}(i)$ and out-degree $d_{out}(i)$ of a node i are defined as:*

$$d_{in}(i) = \sum_{j=1}^{n} a_{ji} \quad , \quad d_{out}(i) = \sum_{j=1}^{n} a_{ij} \tag{13}$$

and give a measure for the interconnectedness of the node i in a directed graph $G(V, E)$. The two degrees are equal for directed graphs.

The size of a node i can be defined analogously to its interconnectedness in terms of the value in- and out-degree.

Definition 5. *The value in- and out-degree of a node are defined as:*

$$vdc_{in}(i) = \frac{\sum_{j=1}^{n} w_{ji}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{kj}} \in [0,1] \tag{14}$$

$$vdc_{out}(i) = \frac{\sum_{j=1}^{n} w_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{jk}} \in [0,1] \tag{15}$$

and give a measure for the size of the node. The value in-degree is a measure of the liabilities of a node, while the value out-degree is a measure of its assets.

A quantity that can be used to characterise a network is its average path length. The average path length of a network is defined as the average length of shortest paths for all pairs of nodes $i, j \in V$. Another commonly used quantity to describe the topology of a network is the clustering coefficient, introduced by Watts and Strogatz (1998) in their seminal work on small-world networks. Given three nodes i, j and k, with i lending to j and j lending to k, the clustering coefficient can be interpreted as the probability that i lends to k as well. For $i \in V$, the number of opposite edges of i is defined as

$$m(i) := |\{j,k\} \in E : \{i,j\} \in E \text{ and } \{i,k\} \in E| \tag{16}$$

and the number of potential opposite edges of i as

$$t(i) := d(i)(d(i) - 1) \tag{17}$$

where $d(i) = d_{in}(i) + d_{out}(i)$ is the degree of the vertex i. The clustering coefficient of a node i is then defined as

$$c(i) := \frac{m(i)}{t(i)} \tag{18}$$

and the clustering coefficient of the whole network $G = (V, E)$ is defined as

$$C(G) := \frac{1}{|V'|} \sum_{i \in V'} c(i) \tag{19}$$

where V' is the set of nodes i with $d(i) \geq 2$. The average path length of the whole network can be defined for individual nodes. The single source shortest path length of a given node i is defined as the average distance of this node to every other node in the network.

It is possible to distinguish between a number of networks by looking at their average path length and clustering coefficient. One extreme type are regular networks which exhibit a large clustering coefficient and a large average path length. The other extreme are random networks which exhibit a small clustering coefficient and a small average path length. Watts and Strogatz (1998) define an algorithm that generates a network which is between these two extremes. They could show that *"small-world networks"* exhibit both a large clustering coefficient and small average path length. A large number of real networks, such as the neural network of the worm Caenorhabditis elegans, the power grid of the western United States, and the collaboration graph of film actors are small-world networks. From a systemic risk perspective, small-world networks are interesting as it is reasonable to assume that the short average path length and high clustering of small-world networks make them more vulnerable to contagion effects than random or regular networks. Small-world networks can be created by using the algorithm defined in Watts and Strogatz (1998). The starting point for this is a regular network of N nodes where each node is connected to its m neighbours. The algorithm now loops over all links in the network and rewires each link with a probability β. For small values of β (about 0.01 to 0.2), the average path length drops much faster than the clustering coefficient, so it is possible to have a situation of short average path length and high clustering. A small-world network is shown on the left side of Figure (2) with $N = 50$, $k = 4$, $\beta = 0.05$.

Scale-free networks are another interesting class of networks. They are characterized by a logarithmically growing average path length and approximately algebraically decaying distribution of node-degree (in the case of an undirected network). They were originally introduced by Barabási and Albert (1999) to describe a large number of real-life networks, such as social networks, computer networks and the World Wide Web. To generate a scale-free network one starts with an initial node and continues to add further nodes to the network until the total number of nodes is reached. Each new node is connected to k other nodes in the network with a probability that is proportional to the degree of the existing node. When thinking about financial networks, this preferential attachment resembles the fact that larger and more interconnected banks are generally more trusted by other market participants and therefore form central hubs in the network. On the right-hand side of Figure (2) a scale-free network with $N = 50$ and $k = 2$ is shown. A number of empirical studies find real-world interbank markets to be scale-free (see, for example, Cajueiro and Tabak (2007), Iori et al. (2008)). Cont and Moussa (2009) show that a scale-free interbank network will behave like a small-world network when Credit Default Swaps (CDS) are introduced. In this sense, a CDS acts as a "short-cut" from one part of the network to another. This paper therefore focuses on these three classes of networks (random, scale-free and small-world) to analyze their impact on systemic risk through contagion effects.

3.4. Model Parameters

There are 18 model parameters that control the numerical simulation. If not stated otherwise, numerical simulations were performed with the parameters given in this section. The simulations were performed with $N = 100$ banks and $\tau = 1000$ update steps each. Note that the simulation results do not change if the number of banks is increased. It has to be ensured, however, that the number is sufficiently large for differences in the network topologies to become significant enough to be visible in the simulation results. The

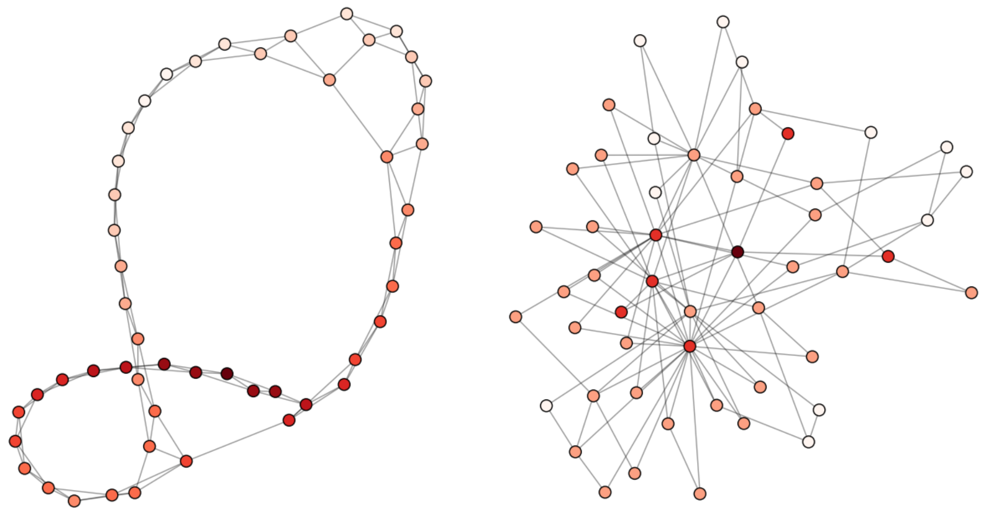


Figure 2: On the left: a small-world network that was created using the algorithm of Watts and Strogatz (1998) with $N = 50$, $k = 4$ and $\beta = 0.05$. On the right: a scale-free network that was created using the methodology introduced in Barabási and Albert (1999) with $N = 50$ and $m = 2$. The colour is an indication for the single source shortest path length of the node and ranges from white (large) to red (short).

number of update steps has to be large enough for the system to reach a steady state from where the results change only little. Every simulation was repeated `numSimulations=100` times to average out stochastic effects. The interest rate on deposits was chosen to be $r^d = 0.02$ and the main refinancing rate as $r^b = 0.04$, which resembles the situation in the eurozone prior to the crisis. The required reserve rate is $r = 0.02$ which is in line with legal requirements, for example, in the eurozone. The interbank connection level for random graphs is denoted as `connLevel`$\in [0, 1]$. At a `connLevel=0` there is no interbank market and at `connLevel=1` every bank is connected to every other bank. If not stated otherwise, a connection level of 0.2 is used. For scale-free networks the parameters $m = 1, 2, 4, 10$, and for small-world networks the parameters $\beta \in [0.001, 0.1]$ were used.

Two sets of parameters are used to describe the influence of the real economy on the model. The first set is the probability that a credit is returned successfully, $p_f = 0.97$ (3% of the credits will default). The return for a successful returned credit is taken to be $\rho_f^+ = 0.09$ and in case a credit defaults, the negative return on the investment is $\rho_f^- = -0.05$. The choice of parameters again resembles the situation in the eurozone and will sometimes be referred to as "normal" parameters. As "crisis" parameters, $\rho_f^+ = 0.97$ and $\rho_f^- = -0.08$ were used. This implies that banks have larger losses on their risky assets in times of crises. To plan their optimal portfolio, the banks have an expected credit success probability p_b and expected credit return ρ_b^+. It is assumed that these expected values correspond to the true values from the real economy. The optimal portfolio structure and volume of a bank also depend also on its risk aversion parameter θ. For each bank, $\theta \in [1.67, 2.0]$ was chosen randomly to allow for heterogeneity in the banking sector. For $\theta < 1.67$, and given all other chosen parameters, portfolio theory would imply that banks hold no risk-free assets. The value of the factor of constant relative risk aversion is subject to an ongoing debate, even though a value greater than 1 is well established (see for instance the discussion in Ait-Sahalia and Lo (2000)).

Deposit fluctuations are modelled as a random walk:

$$D_t^k = (1 - \gamma^k + 2\gamma^k x)\underline{D}_{t-1}^k \tag{20}$$

with $\gamma^k = 0.02$ (in "normal" times) and $\gamma^k = 0.1$ (during a "crisis" period) can be interpreted as a scaling parameter for the level of deposit fluctuations and x being a random variable with $x \in [0, 1]$. The fraction of a bank's investments that the central bank accepts as collateral is set to $\alpha^k = 0.8$, assuming that banks invest only in assets which have a good rating. It is assumed that shareholders can find more profitable investment opportunities than the deposit facility of the central bank and will thus push for banks to pay out as much of the end-of-period-profits as possible. Thus, all excess liquidity at the end of the period (i.e. after banks have satisfied their optimal investment plans and engaged in interbank lending) will be paid out to the banks' shareholders.

4. Results

The three key questions that this paper answers are: (i) Are some network structures more resilient to systemic risk than others? (ii) Can central banks stabilize interbank markets? And (iii) How does a common shock to the banking capital of all banks interact with the counterparty risk externality? Each of these questions is addressed in turn.

4.1. Interbank network structure and financial stability

In Figure (3) the impact of different network topologies on financial stability in times of crisis and normal times is shown for random topologies with varying connectivity. It can be seen that the difference in network topology is not significant during normal times. In times of crisis, however, the different levels of interconnectedness come into play. Figure (3) also confirms the result of Nier et al. (2008), who show that the relationship between the level of interconnectedness on interbank markets and financial contagion is non-monotonic. If there is no interconnectedness, banks are unable to achieve short-term funding on the interbank market and become insolvent. When the connection level is increased to 0.2, more banks survive in the long run.

However, when the connection level is increased even further, the system enters the contagious regime where an initial default can reach a large number of banks and lead to widespread default cascades. This is precisely the tipping point behaviour that Gai and Kapadia (2010) show in a static setting generalized to a dynamic framework. Being in the contagious regime implies that an initial shock (e.g. an idiosyncratic shock to a single bank leading to its default) can spread to the entire system and cause a system-wide crisis. Being in the non-contagious regime implies that any initial shock will be contained within a small fraction of the system.

Figure 4 shows that contagion effects tend to be larger in random networks than in small-world networks, where, in turn, contagion effects tend to be larger than in scale-free networks.[10] This implies that analyses which are conducted with static random networks can overestimate contagion effects. Empirical analyses of interbank networks find them to be of a *"money center type"*, where a small number of large banks is very highly interconnected and a large number of banks is very little interconnected. This is good news in the light of the results in this section as it implies that contagion effects will be limited.

For increasing levels of interconnectedness in random networks, it can be seen from Figure (3) that there exists a tipping point, where the networks become endogenously instable. To better understand this, the interbank

[10] In accordance with the results of Figure 3, the effect of different network topologies is negligible in normal times.

loan volume is depicted in Figure (5) for random and scale-free networks. As Ladley (2011) argues, the knife-edge property of interbank markets requires shocks to be small in order to exihibt a stabilizing effect. Figure (5) shows an increase in interbank market volume until a tipping point, where the amount of interbank loans becomes large and contagion effects dominate. This in turn leads to an increasing number of insolvencies that spread more easily in the system if the level of interconnectedness increases.

4.2. The effect of central bank policy

To answer the question of what impact central bank activity has on financial stability, I first varied the level of collateral α^k that is accepted by the central bank in order to provide liquidity to banks. For $\alpha^k = 1.0$ the central bank will accept all assets of commercial banks as collateral, while for $\alpha^k = 0.0$, no assets will be accepted. Thus, α^k is used as a parameter to determine the fraction of assets that are of sufficient quality to be accepted as collateral. Banks will obtain liquidity for the amount of collateral that they can deposit at the central bank. In Figure (6) it can be seen that a significant stabilizing effect from the liquidity provision by the central bank is obtained above a certain threshold. The precise value of the threshold depends on the specific parametrization and network structure used, but its existence is confirmed for all simulations conducted. However, this stabilizing effect is non-linear in α^k which implies that, on the one hand, even slight changes in the collateral requirements can have significant stabilizing effects if performed around the critical value. On the other hand, even large changes can have very little effect if performed away from the critical value. The effect on the number of active banks is similar for both, the normal and the crisis scenario. In the bottom panel of Figure (6) the impact of the collateral requirements on the volume of interbank loans is displayed. It can be seen that abundant provision of central bank liquidity will lead to a crowding-out effect on interbank liquidity. It can further be seen, that a large amount of interbank liquidity is correlated with high financial instability when looking at the top panel of Figure (6). This is precisely the knife-edge property of interbank markets: if the exposures amongst banks are too large, an initial knock-on effect will be amplified in the system.

4.3. The impact of common shocks on financial stability

To understand the impact of different forms of systemic risk on financial stability, Figure (7) compares two different types of shocks. In the case of pure interbank contagion, the largest bank in the system is selected and exogenously sent into default (caused, for example, by a idiosyncratic shock to the banking capital). In the event of a common shock, all banks suffer a simultaneous loss of $x\%$ on their banking capital. The impact of each of these shocks on the remaining number of active banks in the system is depicted at the top of Figure (7). To analyze the impact the different shocks have on the liquidity provision in interbank markets, at the bottom, Figure (7) shows the interbank market volume. When a common shock hits the system, banks with insufficient equity will go into insolvency. While this might be only a small number of banks, a larger number of banks become more vulnerable to deposit and asset return fluctuations. As was seen in Figure (refResults:Fig3), shocks that exceed a certain threshold will lead to an increased number of insolvencies in the system. When banks become more vulnerable, this threshold is reached more easily and the whole system remains unstable as long as the volume on the interbank market (and hence the magnitude of possible shocks) leads to increased insolvencies. When the crisis hits, the volume of interbank transactions drops until it has reached a level where the endogenous deposit and asset return fluctuations will not lead to an increased number of insolvencies. Comparing the case of common shocks to the case of interbank contagion, it can be seen that, while the impact of a common shock on the number of active banks is more severe than in the contagion case, the opposite holds true for interbank market liquidity. The pure contagion case has a substantial impact on interbank market liquidity, which, on the other hand, implies a smaller size of shocks due to endogenous fluctuations.

5. Discussion

The model presented in this paper gives rise to a number of interesting questions that could be addressed, but are beyond the scope of the present paper.

Varying required reserves. This paper considers the effect of central bank policy on financial stability. In particular, it analyzes how central bank liquidity provision can prevent widespread default cascades. However, liquidity provision through open market operations and via the standing facilities is not the only tool available to central banks. Another possibility would be to vary the required deposit rate. Such a variation between $r = 0.01$ and $r = 0.25$ is shown in Figure (8). It can be seen that varying reserve requirements has a positive but small effect on financial stability. This is not surprising since the required reserves in this model act as a constraint on the available risky investment (see Equation (1). Less risky investment almost mechanically implies a higher level of financial stability.

Introducing new banks. The model as presented in this paper does not allow the entry of new banks in the course of the simulation. As a measure of financial stability, I used the number of active banks over time. A regime of financial instability is characterized by a relatively larger number of bank insolvencies at every point in time. While this simple measure captures the behaviour of banking crises, it is reasonable to ask how the results would change if new banks were allowed to enter the model. One way to introduce new banks into the system is by taking competition amongst banks explicitly into account. As long as there are banks which make profits (i.e. are able to pay out dividends), there is an incentive for another bank to enter the market and compete for those profits. In the present model, however, banks are myopic profit maximizers and not in direct competition with each other. This could be because the banks in this paper have regional monopolies.

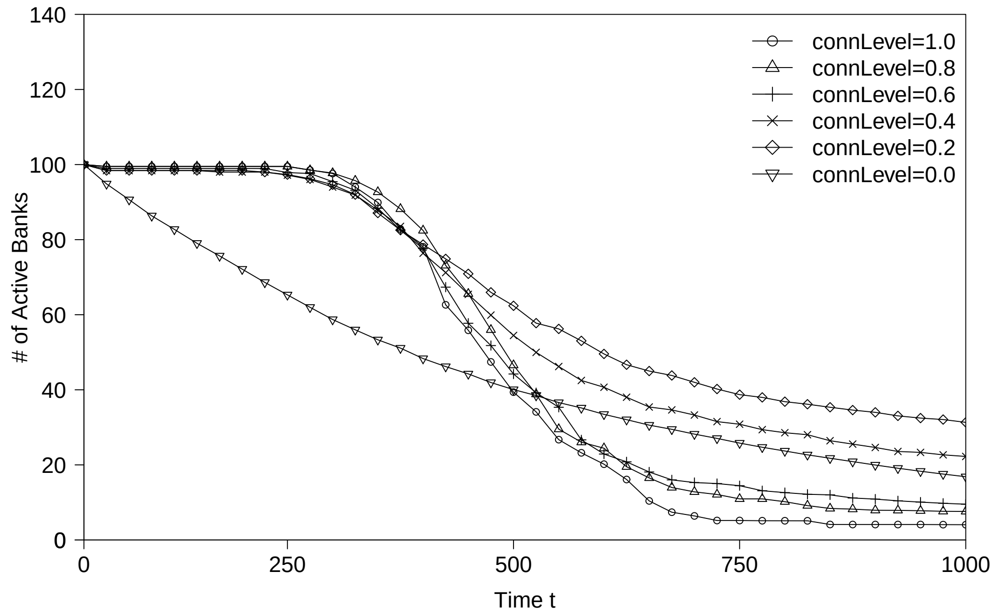
In order to quantify the effect of new banks entering the system nonetheless, Figure (9) uses a crisis scenario and a scale-free network with $m = 10$. In contrast to Figure (4), however, I now allow a bank to enter each k update steps. The simulations start with $K_0 = 0, 80, 50, 100$ active banks and a new bank is added regularly until at $t = 1000$ there would be 100 active banks if there is no insolvency. This simple method makes it possible to characterize the long-run state of the model. It can be seen from Figure (9) that the effect of new bank entries diminishes over time and that a steady state will be achieved. Which steady state is achieved, however, depends on the number of bank entries, where more bank entries yield a higher number of banks in the steady state. The intuition behind this result is that default cascades can wipe out a larger part of the financial system when there are more banks initially. While this result gives some intuition about the impact of new banks entering the system, it cannot fully substitute for an analysis of bank competition. Such an analysis, however, will require significant improvements to the microfoundation of banking behaviour.

6. Conclusion

This paper analyzes different forms of systemic risk in a dynamic multi-agent simulation with portfolio-optimizing banks that engage in bilateral interbank lending. Three key results are obtained. First, complementing the existing literature, which analyzes static interbank networks only, this paper shows that the interbank network structure does have a substantial impact on financial stability only in times of distress. Second, this paper also incorporates the central bank and shows that central bank intervention can alleviate financial distress and liquidity shortages on interbank markets in the short run. Finally, this paper shows that banks become more vulnerable to endogenous fluctuations and occasional idiosyncractic insolvencies when a common shock strikes the entire banking system. While interbank contagion drains interbank liquidity and can thus be alleviated by central bank liquidity provision, an abundance of liquidity will lead to even further insolvencies when systemic risk manifests itself as a common shock.

The results in this paper therefore also shed a novel light on the emergency measures undertaken by central banks at the height of the 2007/2008 financial crisis. While abundant central bank liquidity provision was necessary to ensure the functioning of interbank markets in the short run, the long-run effects of this liquidity provision will be significantly smaller and central banks inadvertently increase the risk of the financial system entering into a *contagious regime*.

From the perspective of policymakers, this paper provides evidence that the topology of the interbank network has to be taken into account. The interbank network topology, however, is highly dynamic and varies from day to day. This implies that further analyses of this dynamic behaviour are necessary in order to understand the full impact of the network topology on the propagation of shocks.



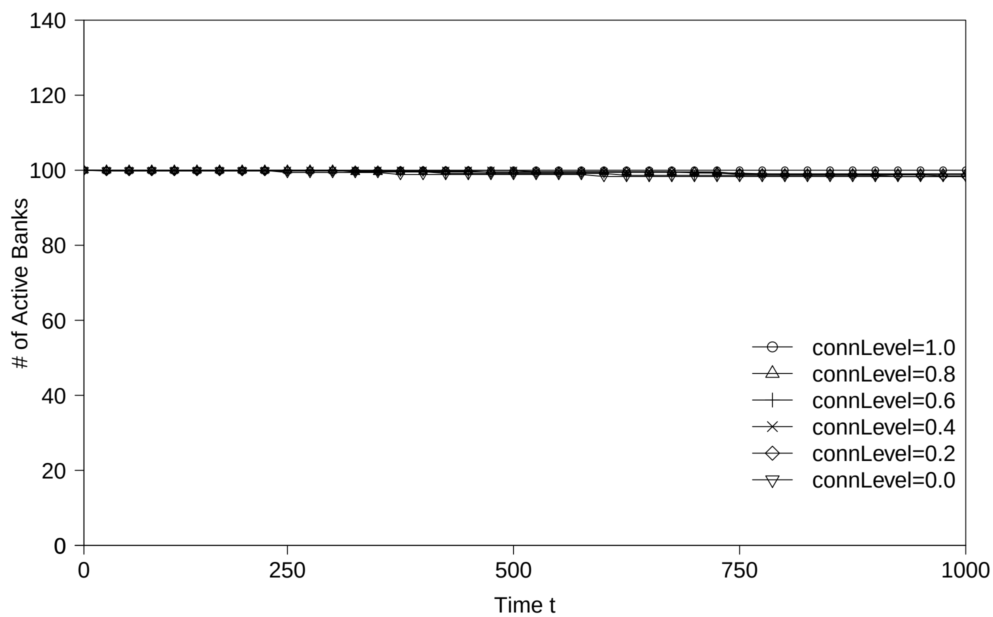
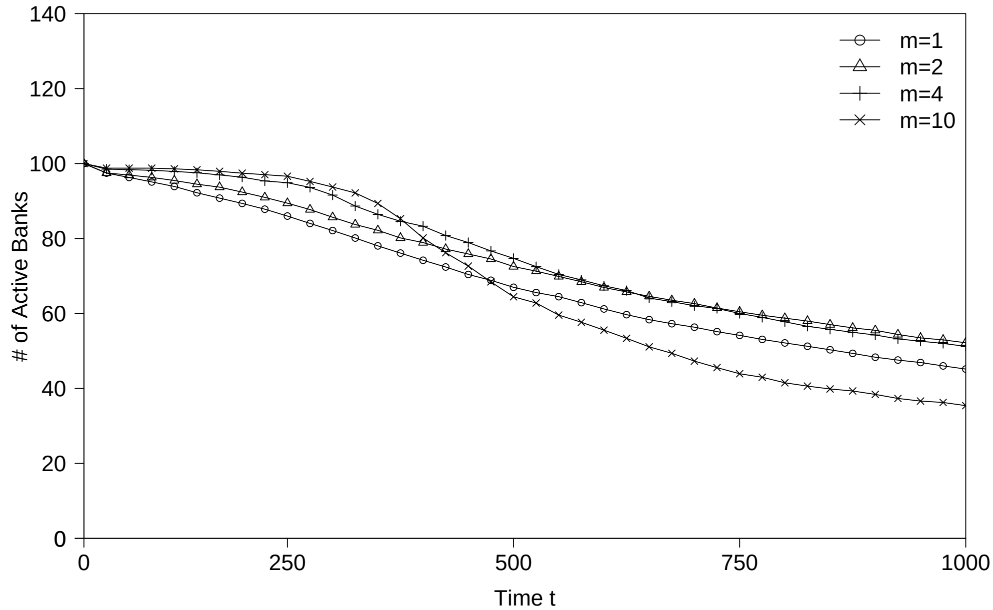


Figure 3: The effect of different network topologies on financial stability. Top: crisis scenario and random topology. Bottom: normal scenario and random topology. Connection levels of `connLevel`$= 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ were used.



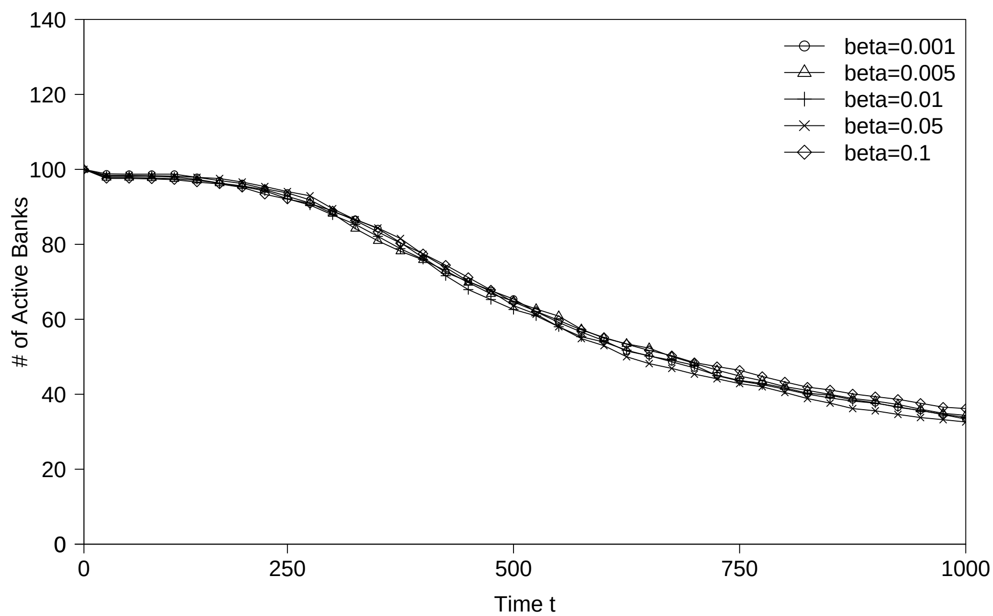
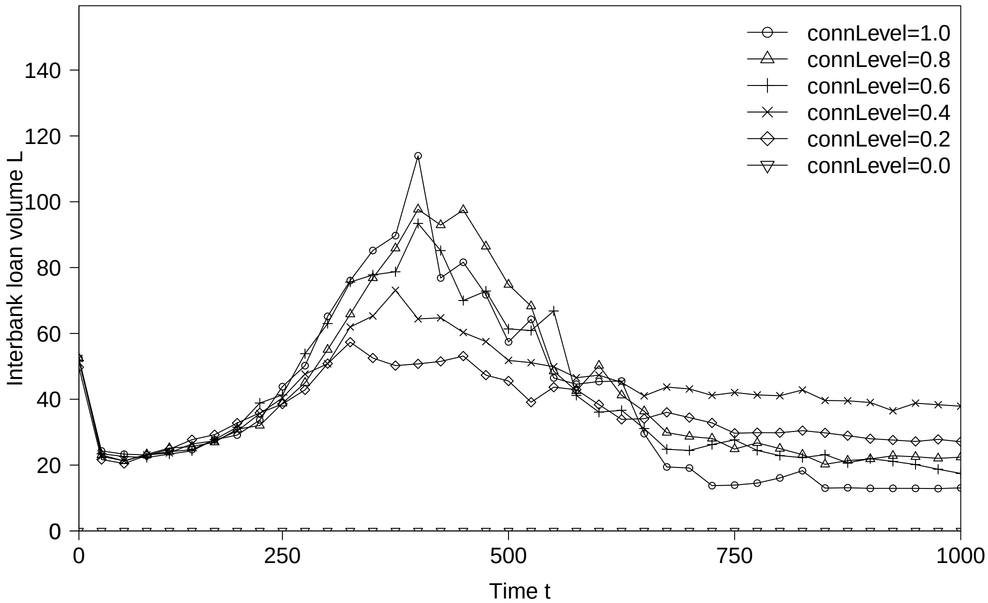


Figure 4: The effect of different network topologies on financial stability. Top: crisis scenario and scale-free (BA) network with $m = 1, 2, 4, 10$. Bottom: crisis scenario and small-world (WS) network with $\beta = 0.001, 0.005, 0.01, 0.05, 0.1$.



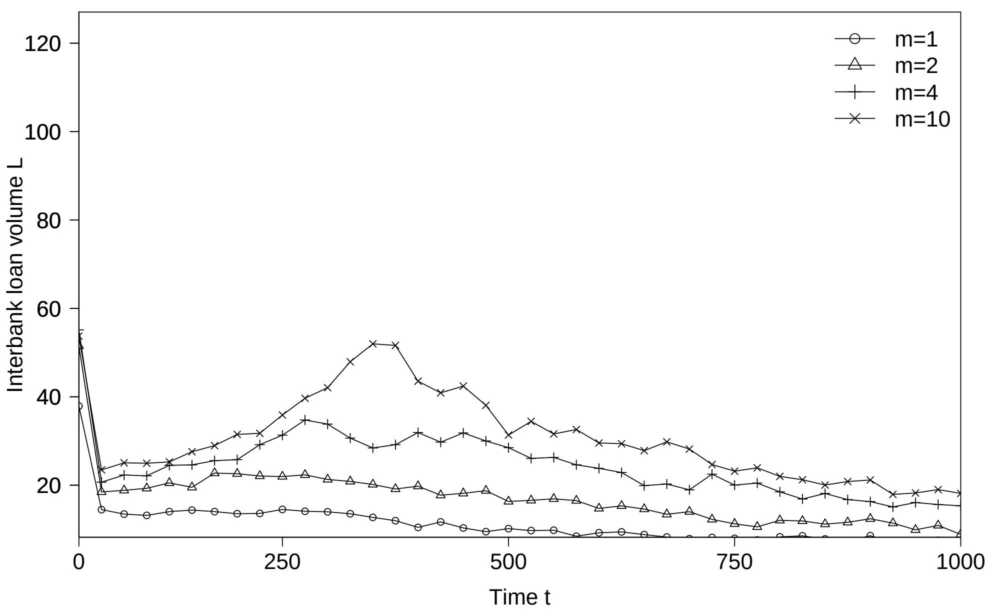


Figure 5: The effect of different network topologies on interbank loan volume. Top: Crisis scenario and random topology, with connection levels of `connLevel`$=$ $0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. Bottom: Crisis scenario and scale-free network with $m = 1, 2, 4, 10$.

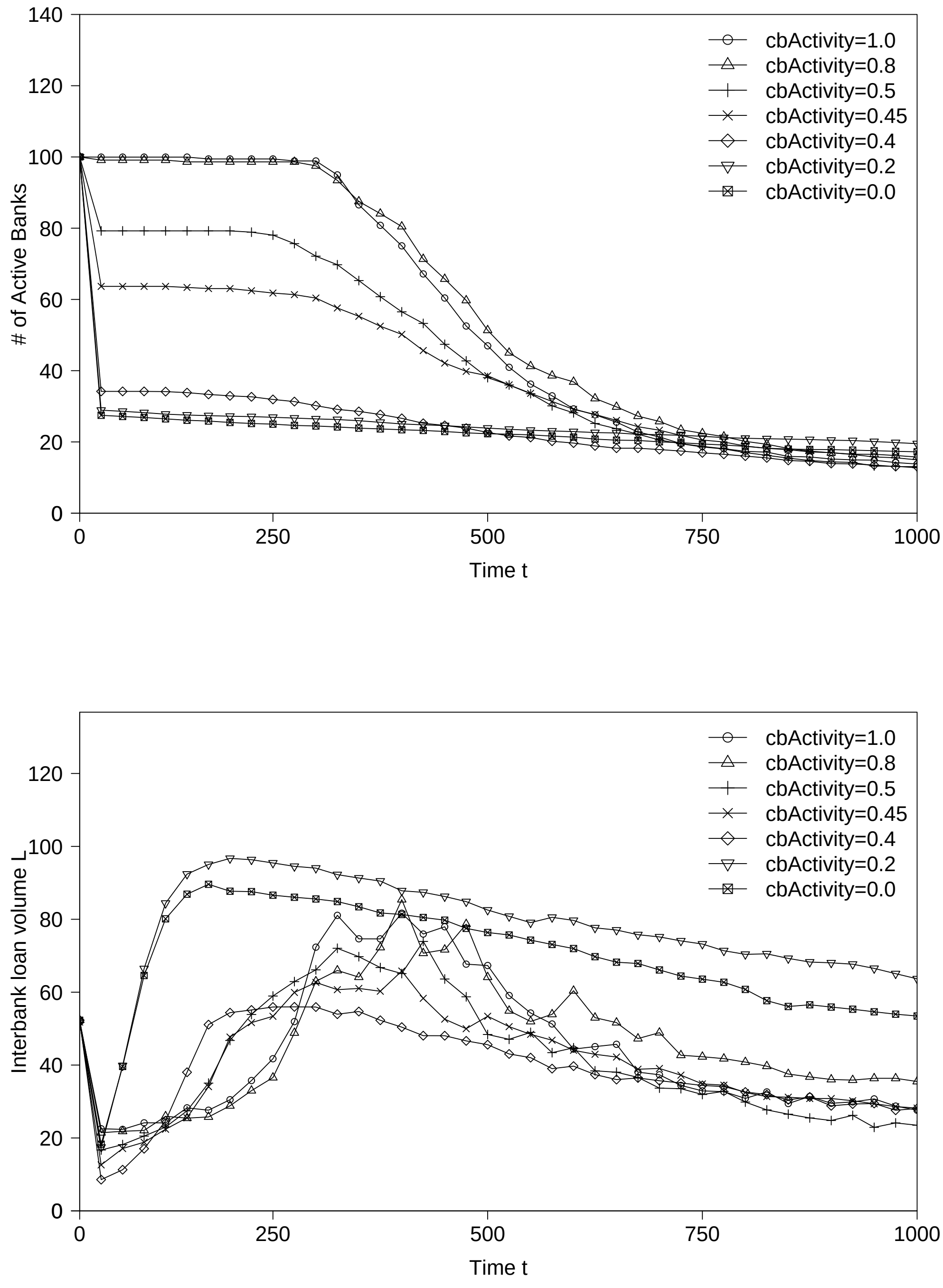
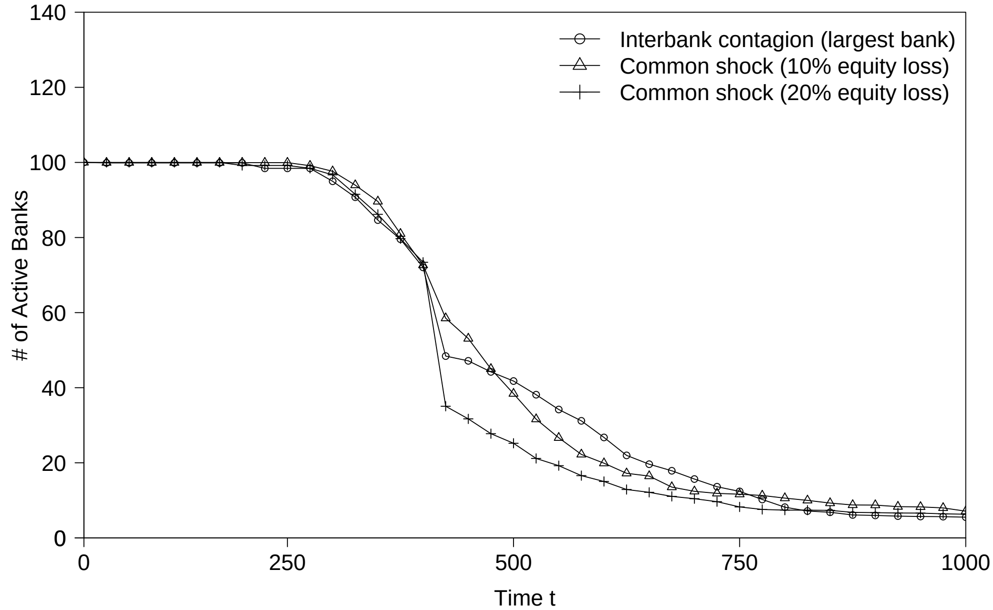


Figure 6: The effect of central bank activity for different scenarios. Top: Number of active banks over simulation time for a random network with connectivity of 0.2. Bottom: Interbank loan volume over simulation time for a random network with connectivity of 0.2. The central bank activity α^k varied between $\alpha^k = 0.0$ and $\alpha^k = 1.0$.



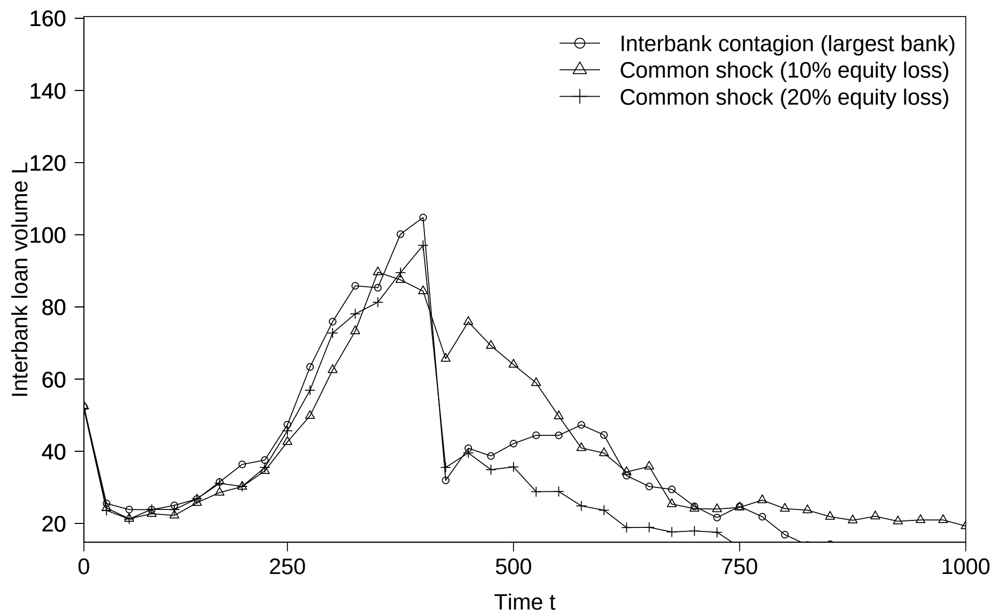


Figure 7: The impact of different forms of systemic risk on financial stability and interbank loan volume in a crisis scenario with random network and a connection level of 0.2. Top: number of active banks over time. Bottom: interbank loan volume over time. Shocks were exogenously applied at $t = 400$.

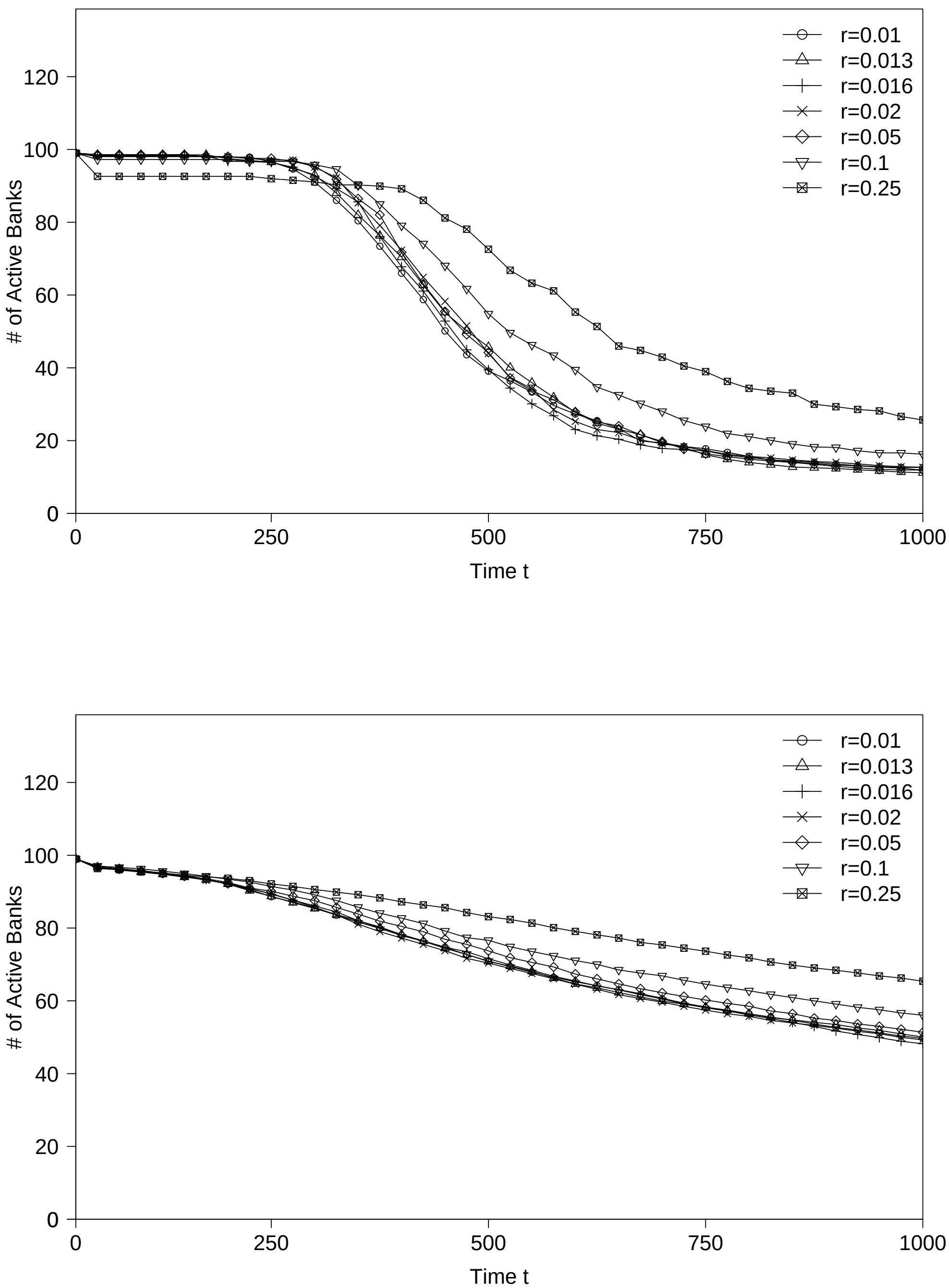
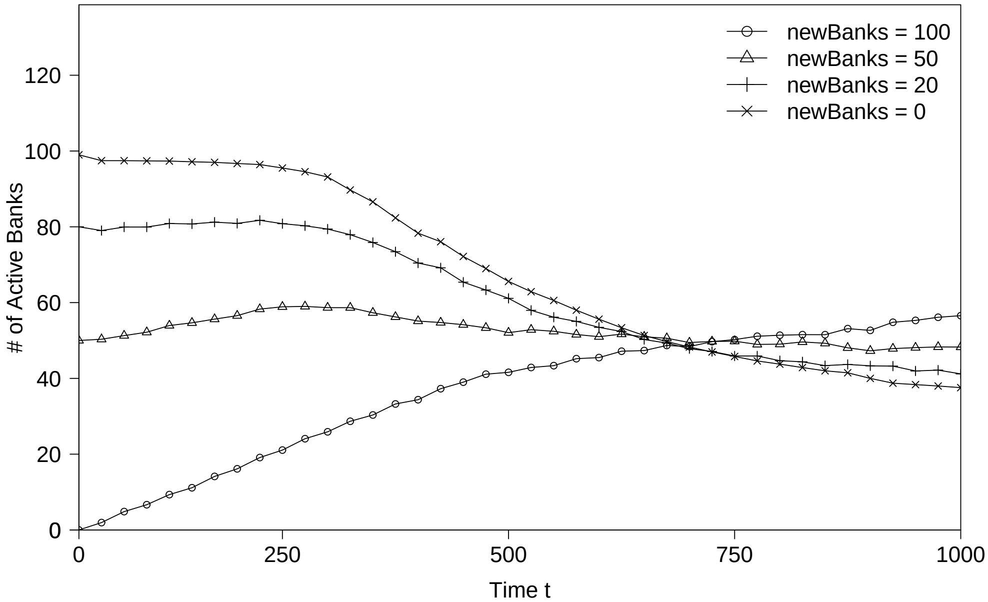


Figure 8: Number of active banks when reserve requirements are varied. A crisis scenario with central bank activity is used. Top: Highly connected random interbank network with connectivity level of 0.5. Bottom: scale free network with $m = 4$.



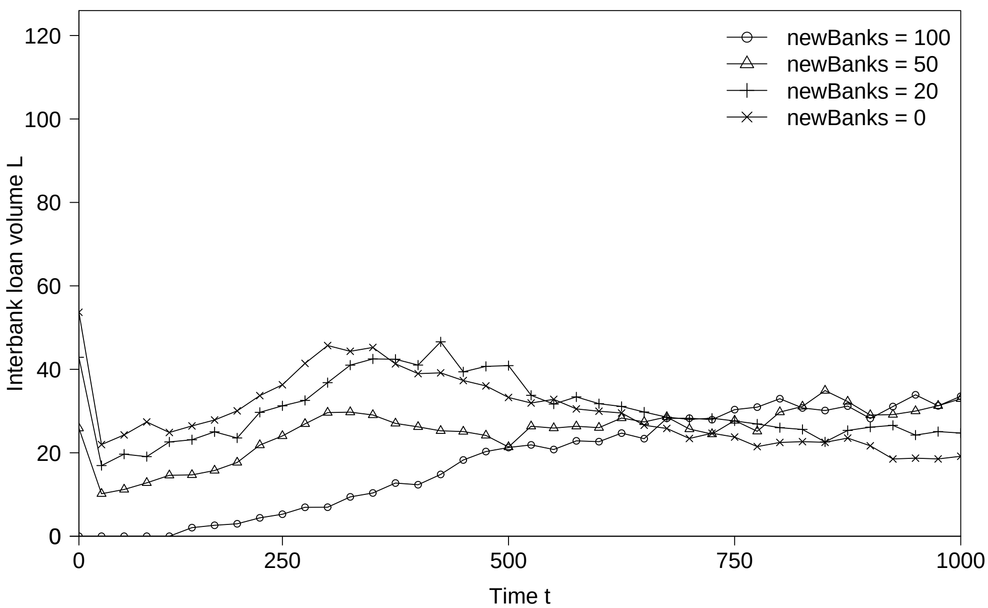


Figure 9: Number of active banks and volume of interbank lending when new banks enter the system.

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